Numerical analysis of hydraulic fracturing process in wellbores; A three dimensional mixed mode study

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Abstract
The hydraulic fracturing operation is a suitable method for stimulating and increasing the productivity of oil and gas wells. Since the hydraulic fracturing is essentially a process of crack growth in the wellbores and the reservoirs formations, it is preferred to investigate this method using the concepts of rock fracture mechanics. In practice, because of heterogeneity and anisotropy of rock layers and formations and also the magnitude and the direction of in situ stresses, the created hydraulic fractures may grow under mixed mode tensile-shear loading. Thus in this paper, the stress intensity factors of a semi-circular crack in the wall of a vertical wellbore were determined by means of various 3D finite element analyses in the ABAQUS code. In the analyzed models, the influence of the initial crack direction relative to the in situ principal stresses on the stress intensity factors K_I and K_{II} were investigated. It was shown that by changing the angle between the crack plane and the maximum horizontal principal stress (σ_2), the mode I component decreases and the corresponding mode II stress intensity factor (K_{II}) increases. However, the effect of K_I was more pronounced than K_{II} in deformation of crack during the hydraulic fracturing process. A design mixed mode fracture curve was also presented for theoretical estimation of the minimum required pumping pressure for any mixed mode loading situation in the hydraulic fracturing.

1. Introduction
During the past decades, many research studies have been conducted in the field of hydraulic fracturing [1]. This technique is basically used for increasing the productivity of oil and gas wellbores. In this method an artificial fracture is induced in the wall of well by applying the fluid pressure which is pumped by especial devices [2]. The created fractures are then extended into targeted rock formations and hence increase the productivity of wells [3]. Fig. 1 shows a schematic representation of this process [4]. Since the hydraulic fracturing is essentially a process of crack growth in the wellbores and the reservoirs formations, it is essential to investigate this method using concepts of rock fracture mechanics. In this discipline, the state of stress/strain field for any types of crack deformations including opening (mode I), in-plane sliding (mode II) and out-of-plane sliding (mode III) is defined by the stress intensity factors. In most of the previous analytical and theoretical fracture mechanics models it is assumed that the initiation and then growth of hydraulic fracture is occurred under pure tension or pure mode I condition [5]. However, because of heterogeneity and anisotropy of rock layers and formations, the created hydraulic fractures in practice may grow under mixed mode loading. In these cases, the influence of other fracture modes including in and out of plane sliding can affect the fracture process in addition to the crack opening mode. Meanwhile, the hydraulic fractures in inclined wells generally initiate and propagate under mixed mode loading. Therefore, for a more accurate analysis of hydraulic

fracturing, it is important to investigate this procedure using mixed mode fracture models [6]. For understanding the mixed mode crack growth behavior during the hydraulic fracture and for controlling the fracture path, the stress state in front of the growing crack should be studied. The stress intensity factors are the main parameters for describing the crack growth behavior during the hydraulic fracturing process.

The magnitude and the direction of in situ stresses at great depths can affect strongly the fracture behavior of rock masses around the wellbores. Fig. 2 shows three principal stresses that are applied to a rock element at a great depth. The vertical principal stress component (σ_1) is induced from the weight of upper rock masses and the two horizontal stresses of σ_2 and σ_3 are the maximum and minimum horizontal principal stresses, respectively induced from the tectonic plate and the earth layers reactions [7].

In this paper, the fracture parameters of a semi-circular crack induced due to perforation in the wall of a vertical oil well are determined numerically by means of various 3D finite element analyses. In the analyzed models, the influence of initial crack orientation with respect to the direction of horizontal principal stresses is investigated. It is shown that for many of the considered situations, the hydraulic fractures grow in a mixed mode manner and hence the sliding mode stress intensity factor also affects the fracture behavior in addition to the mode I stress intensity factor.

2. Numerical modeling of hydraulic fracturing process

Fig. 3 shows the 3D geometry and loading configuration of a reservoir rock formation having a vertical well which is subjected to three in–situ stresses of σ_1, σ_2 and σ_3 and an internal pressure of P_f. A semi circular crack perforated in the wall of well and perpendicular to the borehole is also considered that generally makes an angle σ with respect to the direction of σ_2 as shown in Fig. 4.

For analyzing the fracture behavior of the cracked wellbore under the illustrated loading condition a 3D finite element model of the well and rock formation was created in the ABAQUS finite element code. A cube of 2m length was considered as the formation rock and, as shown in Fig. 5 a cylindrical hole with diameter of 20 cm was created in the middle of cube to simulate the well. Also the radius of the semi circular crack perforated in the wall of well was chosen equal to 4 cm. The elastic material properties of a typical reservoir formation rock as E = 10 GPa and ν = 0.25 reported by Ingraffea et al. [8] were also considered in the finite element models. A total number of 11000 solid brick elements were used for creating the finite element model. A large number of fine and singular elements were also used in the first ring of crack front line for producing the square root singularity of stress/strain field. Fig. 5 shows the finite element mesh pattern for the well and a zoomed view of the crack region. The stress intensity factors (K_I and K_{II}) are functions of the applied in situ stresses and the crack geometry as well and can be written as:

$$K_i = f(\sigma_1, \sigma_2, \sigma_3, P_f, \gamma, a) \qquad i = \mathrm{I}, \mathrm{II} \tag{1}$$

where, a is the radius of the semi circular crack. The following stresses given by Ingraffea et al. [8] were also applied to the model: σ_1 = 80 MPa, σ_2 = 70 MPa, σ_3 = 60 MPa and P_f = 60 MPa. A J-integral based method built in ABAQUS was used for obtaining the stress intensity factors K_I and K_{II} directly from software.

3. Results and discussion

The results obtained for mode I and mode II stress intensity factors (K_I and K_{II}) from the finite element analysis have been presented in Fig. 6 for different angles of σ. The normalizing parameter (K_{Ic}) in this figure is the pure mode I fracture toughness which assumes to be a constant material property for any cracked material. For normalizing the obtained stress intensity factors, the reported value of $K_{Ic} = 5$ MPa $m^{0.5}$ for a coal reservoir [8] was used. It is seen from Fig. 6, that when the direction of crack is along the direction of maximum horizontal principal stress (i.e. when the angle σ is zero) the crack is subjected to pure mode I loading ($K_{II} = 0$). By increasing the angle σ from zero, mode II component also appears in the crack deformation. Indeed for non zero crack angles σ, the perforated crack in the wall of wellbore experience a combination of opening-sliding deformation and thus both mode I and mode II components become important in crack growth behavior of the hydraulic fracturing process. However, it is seen from Fig. 6 that the effect of K_I component is more pronounced than K_{II} in the given case. Also by increasing the crack angle σ, K_I decreases and conversely K_{II} increase and at a special crack angle K_I becomes zero while K_{II} is non zero. This angle corresponds to pure mode II (pure shear) loading conditions. For the analyzed model, this condition is achieved typically at an angle $\sigma = 30^o$. After this angle, the K_I component becomes negative and hence the crack faces tend to compress to each other. Thus, for these conditions, the required pumping pressure for opening the crack flanks and growing the hydraulic fracture is increased noticeably.

Since the hydraulic fracturing is in general a process of mixed mode crack growth, for a theoretical evaluation of this process the mixed mode fracture criteria should be used. For example, a theoretical mixed mode fracture curve shown in Fig. 7 has been presented by Erdogan and Sih [9] based on a well known maximum tangential stress criterion. This fracture curve can be used for estimating the required minimum pumping pressure in the hydraulic fracturing process under any combinations of tensile-shear loads including pure mode I and pure mode II and mixed mode I/II conditions. For any mode mixities, by increasing the level of applied pumping pressure, the values of K_I and K_{II} increase linearly and the hydraulic fracture starts to grow when the corresponding values of the mode I and mode II stress intensity factors reach the design curve. Therefore, by estimating the critical stress intensity factors, the corresponding required pumping pressure can be evaluated for any loading conditions. This pumping pressure, as seen from Fig. 7, also depends on fracture toughness of the rock formations in front of the perforated crack in the wall of the wellbore.

4. Conclusions

- Crack growth in hydraulic fracturing process can be occurred generally in mixed mode tensile-shear manner.
- The mixed mode stress intensity factors (K_I and K_{II}) for a vertical well having a semi circular crack in the wall of wellbore subjected to in situ stresses and internal pumping pressure were computed numerically using 3D finite element analyses.
- By changing the orientation of crack with respect to the direction of the maximum horizontal principal stress ($\square_2$), different combinations of mode I and mode II were determined for the investigated model. However the influence of tensile mode was more pronounced than the shear mode.

- For performing an optimum hydraulic fracturing process under mixed mode I/II conditions, the theoretical fracture criterion scan be employed to estimate the minimum required pumping pressure.

5. References

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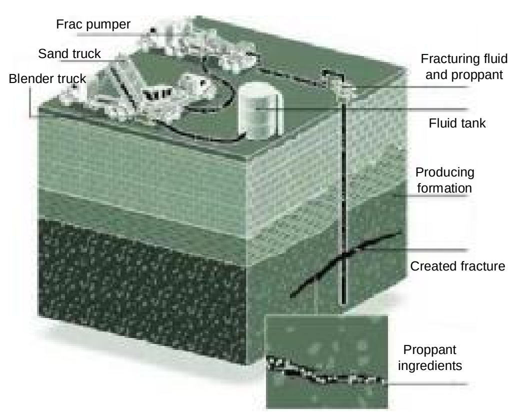

Fig. 1: Schematic representation of hydraulic fracturing process in wellbores [4].

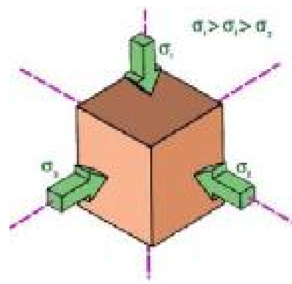

Fig. 2: Principal in situ stresses at great depth applied to a rock element [7].

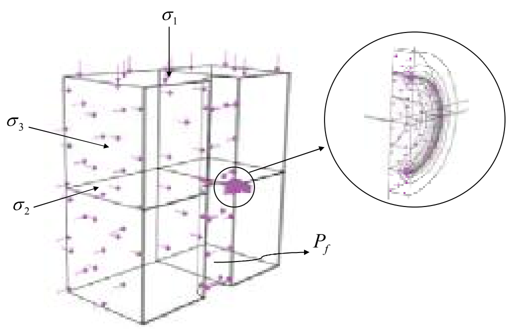

Fig.3: Geometry and loading configuration of reservoir formation containing a cracked vertical wellbore.

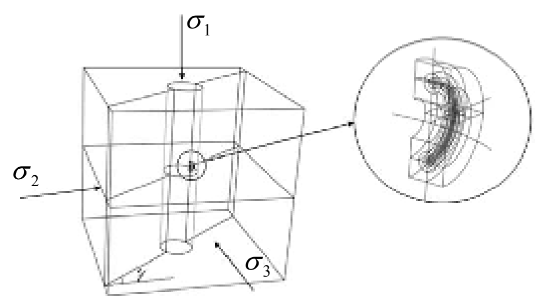

Fig. 4: Inclination angle (□) of a vertical semicircular crack perforated in the wall of vertical well relative to the direction of maximum horizontal principal stress

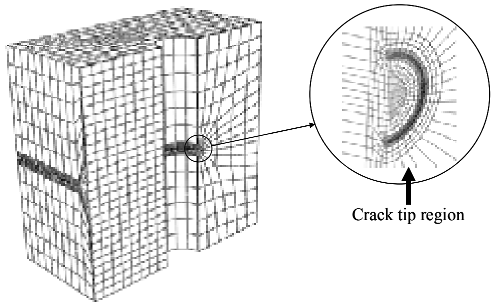

Fig. 5: Finite element mesh pattern created for modeling the reservoir formation, vertical well and a semi circular crack.

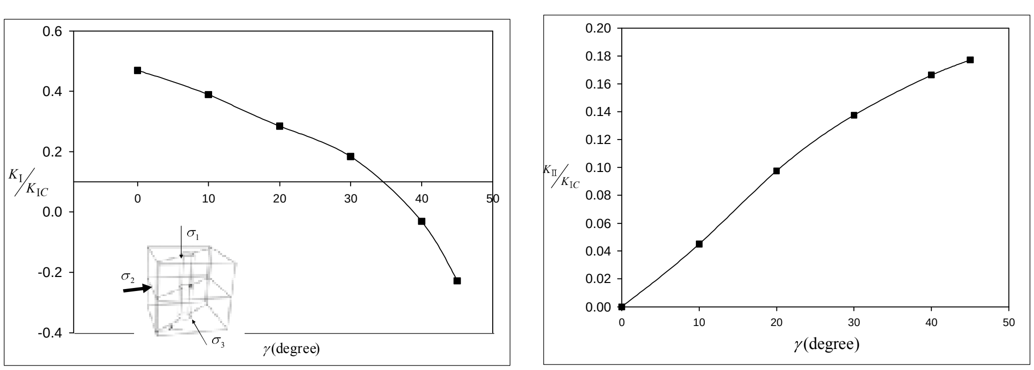

Fig 6: Variations of mode I and mode II stress intensity factors with crack angle ơ for the analyzed crack in the wall of wellbore.

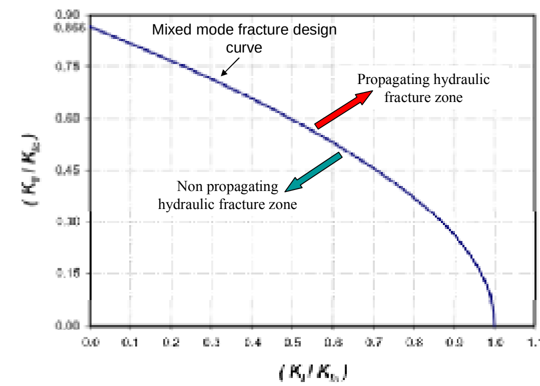

Fig. 7: Mixed mode fracture design curve for estimating the required pumping pressure during a hydraulic fracturing process.